



A hybrid implementation mechanism of tradable network permits system which obviates path enumeration: An auction mechanism with day-to-day capacity control



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ABSTRACT

Akamatsu (2007a,b) proposed a new dynamic traffic congestion control scheme called *tradable network permits*, and demonstrated its efficiency properties for general road networks. To implement tradable permit markets, this paper proposes a novel auction mechanism with capacity control. This mechanism employs an evolutionary approach to achieve a dynamic system optimal allocation of network permits in a computationally efficient manner. We prove that the proposed mechanism has the following desirable properties: (i) truthful bidding is a dominant strategy for each user on each day and (ii) the permit allocation pattern under the mechanism converges to a dynamic system optimal allocation pattern.

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1. Introduction

Congestion pricing is considered an effective economic instrument for managing traffic congestion, and various types of pricing schemes have been proposed since the pioneering work of Pigou (1920) (see Yang and Huang, 2005; Tsekeris and Voß, 2009; de Palma and Lindsey, 2011, for comprehensive reviews and other references). Although these schemes work effectively in ideal situations, almost all of them fail to take into consideration the important fact that *asymmetric information* exists between road managers and road users. For instance, in standard congestion pricing, the road manager requires accurate and detailed demand information (e.g., the desired arrival time and value of time) to calculate optimal toll levels, but it is almost impossible for the manager to obtain such private information. This lack of information may distort toll levels and inevitably result in economic losses.

As an alternative to price-based regulation such as congestion pricing, there is another economic instrument called *tradable permits* scheme, which is a generalization of quantity-based regulation.¹ This scheme directly regulates traffic flows by assigning priority-service permits to road users, which has great potential for not only reducing traffic congestion but also resolving the asymmetric information problem. As an example of such a scheme, Akamatsu et al. (2006) and Akamatsu (2007a,b) proposed a new dynamic traffic congestion control scheme called *tradable network permits*, which does not require

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¹ See, for example, Weitzman (1974) and Laffont (1977) for general discussions on the comparison between price-based and quantity-based regulation in the field of economics.

detailed user information. This scheme consists of two parts: (a) the road manager issues a right (network permit) that allows the permit holder to pass through a bottleneck during a prespecified time period and (b) a trading market is established for network permits differentiated by prespecified time. Under this scheme, queuing congestion can be completely eliminated for each bottleneck by issuing a number of network permits that is less than its capacity. For allocating the permits to users, there are two representative schemes: the *market selling scheme* and the *free distribution scheme*. In the market selling scheme, the road manager sells all the permits to users through the trading markets. In the free distribution scheme, the road manager initially distributes all the permits to users for free according to methods that consider the equity among users. In this scheme, the permits allocated for each user does not necessarily match one's own desired permit. For that case, users can mutually trade permits in the trading markets. With both schemes, the asymmetric information problem is resolved through the trading markets. Furthermore, Akamatsu et al. (2006) demonstrated that under both permit-allocating schemes the equilibrium achieves the most efficient (i.e., Pareto optimal) resource allocation for a single bottleneck, and Akamatsu (2007a,b) extended this property to general networks.²

Although the efficiency of the tradable network permits was established by assuming that a competitive equilibrium can be achieved in the trading markets, no concrete trading mechanism that attains the equilibrium was demonstrated in the above studies. In other words, trading processes were treated as a black-box. Thus, in order to implement tradable network permits, it is necessary to establish a micro-mechanism for the trading markets. It should be noted that in terms of efficiency of resource allocation, the above two permit-allocating schemes are essentially identical. Therefore, as the first step in trading markets design, this paper focuses on the market selling scheme to achieve an efficient allocation as simply as possible.³ In this regard, Wada and Akamatsu (2010) and Wada et al. (2010) designed an auction mechanism for a trading market for a single bottleneck and showed the following: (i) the network permit allocation pattern achieved under the mechanism is efficient and (ii) the mechanism is strategy-proof, which means that a dominant strategy employed by each user is truthful revelation of the value of permits. However, extending the auction mechanism to general networks is not a trivial problem, because a naïve formulation of the problem leads to NP-hardness owing to the complex relationship between link and path.

This paper proposes a novel auction mechanism to implement trading markets on general networks with multiple origin–destination (OD) pairs. Assuming that each user makes a trip from an origin to a destination via a certain path and in a specific time period, we design an auction mechanism that enables each user to purchase a bundle of network permits corresponding to a set of links on the user's preferred path. We first briefly discuss how the Vickrey–Clarke–Groves (VCG) mechanism, which is a benchmark mechanism in auction theory (e.g., Milgrom, 2004), cannot possibly be applied to the trading markets because the combinatorial optimization problem of finding a network permits allocation pattern is NP-hard. To avoid such computational infeasibility, we propose an auction mechanism that is readily implementable. This mechanism employs an evolutionary approach that decomposes the combinatorial optimization problem into two phases, an auction phase and a path capacity adjustment phase, which are repeated on a day-to-day basis. The path capacity is defined as the number of bundles of permits for the path. In the former phase, the manager fixes each path capacity and sells the bundles to users through an ascending auction. In the latter phase, the road manager adjusts each path capacity to an appropriate level by exploiting bundle prices determined in the auction phase. We then prove that the proposed mechanism has the following desirable properties: (i) truthful bidding is a dominant strategy for each user on each day and (ii) the permit allocation pattern under the mechanism converges to an approximate dynamic system optimal allocation pattern in the sense that the social surplus reaches its maximum value when the number of users is large. Finally, we show that the proposed mechanism can be extended to obviate path enumeration by introducing a column generation procedure.

The rest of this paper is organized as follows: Section 2 discusses related work. Section 3 outlines the framework of the tradable network permits scheme and describes assumptions used throughout the paper. Section 4 defines a dynamic system optimal allocation of network permits and discusses the impossibility of applying the VCG mechanism to the trading markets due to NP-hardness. Section 5 presents ideas for a novel auction mechanism that is readily implementable for general networks. Section 6 gives details of the proposed mechanism and clarifies its properties. Section 7 presents an extended mechanism which obviates path enumeration by exploiting a column generation procedure. Section 8 demonstrates the convergence properties of the proposed mechanism through a numerical example. Section 9 concludes the paper.

2. Related work

Our study is mainly concerned with dynamic traffic assignments (DTA), some types of transportation demand management (TDM) schemes (i.e., dynamic congestion pricing schemes and tradable permits schemes), and combinatorial auctions. The first two areas provide an analytical framework for modeling and managing traffic congestion in transportation networks, whereas the third area provides a foundation for constructing an auction mechanism to implement trading markets. In particular, auctions for bundled items with network structure are relevant to our study.

² We can also generalize the theory to include supply side conditions. Specifically, Wada and Akamatsu (2012) proposed a distributed signal control policy based on the tradable network permits, which adjusts a green time proportion by exploiting permit prices.

³ To implement the free distribution scheme, it is necessary to design a micro-mechanism for a double auction market in which trading strategies of users are more complex than those of one-sided auction.

2.1. Dynamic traffic assignment models

Following the successful incorporation of queuing phenomena into transportation network analysis, considerable research into DTA models has been conducted (e.g., Vickrey, 1969; Kuwahara and Akamatsu, 1993; Cascetta, 2001). For instance, departure time choice models have been developed by Smith (1984a), Daganzo (1985), Newell (1987), and Iryo and Yoshii (2007), while dynamic user equilibrium (DUE) models have been developed by Kuwahara and Akamatsu (1993), Smith (1993), Heydecker and Addison (1996), Akamatsu (2001), and Iryo (2011) and many others (see Peeta and Ziliaskopoulos, 2001; Szeto and Wong, 2011, for comprehensive reviews). These studies analyzed the properties of user equilibrium and discussed the effectiveness of dynamic congestion pricing as shown in the next subsection. However, few studies have addressed the asymmetric information problem and the effectiveness of quantity-based regulation for eliminating queues.

2.2. Dynamic congestion pricing schemes

Dynamic congestion pricing is a natural extension of static congestion pricing and is a benchmark TDM scheme to eliminate queuing congestion. Despite its importance, most studies have been limited to simple networks (e.g., a single bottleneck or parallel link) because analyzing DTA models for general networks is usually intractable (e.g., Arnott et al., 1990, 1993; Kuwahara, 2007; Doan et al., 2011). However, there have been some attempts to overcome this difficulty. For example, Ziliaskopoulos (2000) and Nie (2011) studied dynamic marginal cost analyses for system optimal DTA problems with many-to-one (or one-to-many) OD pairs. Yang and Meng (1998) derived an optimal toll based on a time-space network for general networks. Friesz et al. (2007) formulated a dynamic second-best toll pricing problem for general networks as mathematical programs with equilibrium constraints and developed a solution algorithm, but they did not address theoretical questions (e.g., algorithm convergence). In effect, no study has established a theory of dynamic congestion pricing for general networks in which queues arise. Furthermore, implementations of the above schemes unsurprisingly face the difficulty associated with asymmetric information.

To address the asymmetric information problem, some studies have developed evolutionary (trial-and-error) implementation methods for congestion pricing in static settings (Sandholm, 2002, 2007; Yang et al., 2004; Han and Yang, 2009). These methods set toll levels based on *realized* traffic flow patterns. The studies demonstrated that an appropriate adjustment process of route choice (e.g., Smith, 1984b) converges to an equilibrium that minimizes the total transportation cost in the network.⁴ This result relies on the fact that there is an equivalent optimization problem (or a Beckmann-type potential function) for a static user equilibrium. However, the properties of static and dynamic congestion pricing differ because the mechanisms of flow and queuing congestion are totally different. In addition, the DUE model cannot also be reduced to an optimization problem in general. Thus, it is not easy to generalize the methods to dynamic settings. Further, the methods need to set a discriminatory toll to achieve an optimal state when users have heterogeneous costs (e.g., value of time), but information on such heterogeneities cannot be gathered by these methods, which means that this approach is not a panacea for the problem even in static settings.

2.3. Tradable permits schemes for managing traffic congestion

A tradable permits scheme that combines a quantity-based regulation with a market institution has been studied for environmental protection (Montgomery, 1972; Tietenberg, 1980). The capabilities and applicability of this scheme have been increasing, because the emergence of the Internet enables a new market to be established inexpensively. For managing traffic congestion, a few researchers have studied such a scheme as an alternative to congestion pricing. Verhoef et al. (1997) discussed the possibilities of using tradable permits in the various types of regulations for road transport externalities; e.g., vehicle ownership permits, tradable parking permits, and tradable permits in the regulation of road usage. Teodorović et al. (2008) proposed an auction-based congestion pricing, for which drivers wanting to enter a downtown area would have to participate in a downtown time slot auction. Although it formulated the allocation problem for the time slots, their study did not address how to set their prices, which is the core problem for auction mechanisms. Moreover, the existing studies provide some useful insights into tradable permits schemes for managing traffic congestion, but none describes *time-dependent* tradable permits for eliminating *bottleneck congestion*.

In addition, it is worth mentioning that the *tradable travel credit scheme* proposed by Yang and Wang (2011) is superficially similar to but fundamentally different from the tradable network permits scheme.⁵ Basically, under the tradable travel credit scheme, the road manager initially distributes credits to all eligible travelers and predetermines a link-specific credit charge. Credits are freely tradable among the credit holders in a credit market. Yang and Wang (2011) showed that, if the manager can appropriately set the total number of credits and the link-specific credit charges, a desirable traffic flow pattern is

⁴ Unlike Sandholm (2002, 2007), Yang et al. (2004) and Han and Yang (2009) did not explicitly consider an adjustment process. Instead, they assumed that user equilibrium traffic flow patterns are realized for any given temporal link toll pattern, which may imply that it takes time to obtain each equilibrium with the adjustment process.

⁵ Similar schemes of the tradable travel credit were also discussed in Viegas (2001) and Verhoef et al. (1997).

achieved. However, it is apparent that this scheme requires detailed demand information unlike the tradable network permits.⁶ Further, it is fair to say that this scheme is not a quantity-based regulation for managing congestion but rather a redistribution scheme for income. Indeed, the main advantage of this scheme over the standard congestion pricing is the improvement in equity and social acceptability, not a direct reduction in traffic congestion.

2.4. Auction mechanisms for networked items

Since the pioneering work of [Rassenti et al. \(1982\)](#), who proposed airport time slot auctions, there has been a considerable amount of work on combinatorial auctions (e.g., [de Vries and Vohra, 2003](#); [Cramton et al., 2006](#)), which allow bids on combinations of items and thus enhance the economic efficiency when bidders have preferences for sets of items (e.g., spectrum rights, airport time slots, railroad segments, and paths in networks). The most celebrated such auction is the VCG mechanism ([Vickrey, 1961](#); [Clarke, 1971](#); [Groves, 1973](#)). This mechanism is strategy-proof and can achieve allocative efficiency. However, to maintain these properties, it requires the auctioneer to solve complex combinatorial optimization problems to determine the allocation and prices (Vickrey payments). Therefore, the VCG mechanism is computationally intractable in many circumstances, including ours (see Section 4).

In this regard, several authors have clarified how such intractability can be avoided under certain restricted circumstances, in which combinations of items have network structures. [Bikhchandani et al. \(2002\)](#) demonstrated that the VCG outcome can be computed by solving two linear programs in the case that a winner determination problem reduces to a spanning tree problem or a shortest path problem.⁷ [Nisan and Ronen \(2001\)](#) derived the Vickrey payments for a shortest path problem, and [Hershberger and Suri \(2001\)](#) developed an efficient algorithm to compute those payments. However, the auctions cannot be implemented for trading markets because these are reverse auctions that cannot handle multiple buyers (i.e., users).

The studies on bandwidth auctions for communication networks are also related to our study in the sense that those also focus on an allocation problem for a network capacity that is a limited resource (e.g., [Koutsopoulos and Iosifidis, 2010](#)). The studies consider the case in which each bidder (e.g., provider) purchases a quantity of bandwidth over a path in a network. [Lazar and Semret \(1999\)](#) proposed the progressive second price auction for allocating a divisible quantity of bandwidth over a certain path. [Dramitinos et al. \(2007\)](#) proposed a multi-unit Dutch auction, which allocates an indivisible quantity of bandwidth over a certain path. Both of these auction mechanisms can induce truth-telling. However, in contrast to the mechanism that is proposed in this paper, neither takes into account the route choice problem of the bidders (i.e., each bidder is interested in a single fixed path). From the above discussion, we conclude that there is no network auction mechanism that enables us to assign network capacities (i.e., network permits) to multiple users who choose a route in a network, and thus the proposed mechanism is a major contribution of this paper.

3. A system of tradable network permits for transportation networks

3.1. Networks

In this paper, we consider discrete-time dynamic traffic flows on a general network (i.e., a transportation network with general topology). The network consists of a set N of nodes and a set A of directed links. The node set N includes a subset O of origin nodes from which users start their trips, and a subset D of destination nodes at which users terminate their trips. A set of origin–destination (OD) pairs is denoted by W . Each element of A (i.e., each link) is identified by a sequential natural number a .

The time interval $[0, I]$ for which we assign the dynamic traffic flow is fixed. We assume that each OD pair's potential travel demand Q_{od} in the time interval $[0, I]$ is a given constant. The time interval $[0, I]$ is discretized into small intervals of length Δt : each time point is represented by $t = m\Delta t$, where $m = 0, 1, 2, \dots, M$. Each time interval $[t, t + \Delta t]$ is denoted by $t \in T$ and we call this interval *time period* t .

We also assume, without any loss of generality, that each link in a network consists of a free-flow segment and a single bottleneck segment. The travel time to pass through the free-flow segment of link a is a constant t_a . We then assume that travel time t_a is represented by a natural multiplier of Δt (i.e., an integer n_a satisfies $t_a = n_a\Delta t$). The bottleneck of each link is represented by a *point queue model* with constant capacity $\mu_a = \text{vehicles/time interval } \Delta t$.

3.2. Road network manager and users

The model presented in this paper involves two types of agents: a road network manager and road network users. The road manager aims to restrain traffic congestion in the network and maximize the social surplus. To achieve this, the manager regulates the traffic flow rates entering each bottleneck in the network by using time-dependent network permits. The precise definition and setup of the network permit system are described in Section 3.3.

⁶ [Nie \(2012\)](#) pointed out this fact in the context of comparison with tradable permits for emission control: "Suffice it to say here that the information that the government would need to run a mobility credit market is as much as the information required to operate a conventional pricing scheme. Therefore, the mobility credit market does not reduce the administrative burden of the government, unlike in the case of emission control."

⁷ [Bikhchandani et al. \(2002\)](#) also dealt with more general cases.

Within the time interval $[0, I]$, each atomic user $i \in \mathcal{N}_{od}$ (i.e., $|\mathcal{N}_{od}| = Q_{od}$) makes *at most* a single trip in the network from an origin (e.g., residential zone) to a destination (e.g., the central business district). This means that all users do not necessarily make trips, which corresponds to the conventional traffic assignments with elastic demand (see also Section 6.1). The user chooses a destination arrival time period and a path between the origin and destination so as to maximize the utility. Under the system of network permits, each user must purchase a bundle of permits corresponding to a set of links included in the user's chosen path. This implies that choosing a destination arrival time period and a path directly corresponds to purchasing time-dependent network permits in the trading markets.

3.3. Network permits and trading markets

A *time-dependent network permit* is a right that allows the permit holder to pass through a prespecified bottleneck at a prespecified time. In this paper, we assume that the manager can issue time-dependent network permits for all bottlenecks (i.e., links) in the network. This implies that the traffic flow entering the link a at time period t consists only of users who have a permit for link a at time period t and a user without this permit cannot pass through the link in this time period.

Throughout this paper, we assume that the number of permits issued for each link in each time period is equal to or less than the bottleneck capacity of each link in the network. This means that queuing congestion never occurs in the network under this permit-issuing scheme. This is clear from an explanation of the permits: the inflow rate of each link is equal to (or less than) the number of permits issued, and so the inflow rate cannot exceed the capacity of the link, which implies that queuing congestion can never occur at a link.

The permits issued for each link (bottleneck) are put on sale by the road manager. Each user who would like to use a path must purchase a bundle of permits corresponding to the set of links included in the user's preferred path. In the trading markets, the prices and the allocation of time-dependent permits are determined through an auction mechanism. The detailed trading rules are given in Section 6.

It must be admitted that the procedures for trading network permits seem unrealistic at first glance, but implementation of these would become feasible with futuristic vehicles in which an agent software is installed to manage driving, navigation and safety. From this perspective, the mechanism proposed in this paper can be viewed as the protocol of a multi-agent system in which the agent software executes the procedures for trading network permits on behalf of users.

3.4. Dynamic travel costs and user utility in general networks

The transportation cost for a single trip made by a network user consists of schedule cost and travel cost. The schedule cost for user i is the cost due to the difference between the user's desired arrival time period t_i and the actual arrival time period t . The schedule cost is represented by a function $s_i(t, t_i)$ of both destination arrival time and desired arrival time. The travel cost is the monetary equivalent of the travel time for a trip from the origin to the destination. The travel times differ among the paths. The travel time of a path between the OD pair is defined as the sum of travel times of the links included in the path. Note that the travel time of each link a is a constant t_a under the permit system, since there is no queuing. Hence, the travel time T_r for path $r \in R_{od}$ between the OD pair is also constant:

$$T_r = \sum_{a \in A} t_a \delta_{a,r(o,d)}, \quad (1)$$

where $\delta_{a,r(o,d)}$ is a typical element of the path-link incidence matrix for the node pair (o, d) ; it is 1 if link a is on path r connecting the OD pair (o, d) and zero otherwise.

We suppose that each user has a private valuation $v_{i,r}(t)$ for each path r and each destination arrival time period t . This valuation $v_{i,r}(t)$ represents a nonnegative value of trip between OD pair along path r in time period t . For example, to show a correspondence with conventional traffic assignments, we can specify the valuation as

$$v_{i,r}(t) \equiv w_i - (s_i(t, t_i) + \alpha_i T_r), \quad (2)$$

where w_i is a parameter, which is interpreted as the trip utility (or willingness-to-pay) between the OD pair, and α_i is a coefficient that converts travel time into a monetary equivalent.

Each user is assumed to have a quasi-linear utility function (we use the term "payoff" interchangeably with "utility"). Specifically, each user's utility $u_{i,r}(t)$ for path r in time period t is represented as the difference between the private valuation and the permit purchase cost $P_r(t)$ determined at auction:

$$u_{i,r}(t) \equiv v_{i,r}(t) - P_r(t). \quad (3)$$

The permit purchase cost is the total payment for purchasing the bundle of link permits required for traveling along a path and arriving at the destination in a certain time period.

4. Dynamic system optimal allocation of network permits

The objective of an auction mechanism, such as that designed in this paper, is to achieve a network permit allocation pattern that maximizes a social surplus (i.e., dynamic system optimal allocation). The social surplus is defined as the sum of

user's valuations. This excludes user payments to the road manager to purchase permits, because these payments are simply income transfers between the users and the road manager. Thus, we formulate an optimization problem [DSO] of providing the dynamic system optimal allocation of network permits:

$$\max_{(\mathbf{f}, \mathbf{x})} \text{SS}(\mathbf{f}) \equiv \sum_{od \in \mathcal{W}} \sum_{i \in \mathcal{N}_{od}} \sum_{t \in T} \sum_{r \in \mathcal{R}_{od}} v_{i,r}(t) f_{i,r}(t), \quad (4)$$

$$\text{s.t.} \quad \sum_{t \in T} \sum_{r \in \mathcal{R}_{od}} f_{i,r}(t) \leq 1 \quad \forall i \in \mathcal{N}_{od}, \forall od \in \mathcal{W}, \quad (5)$$

$$\sum_{od \in \mathcal{W}} \sum_{i \in \mathcal{N}_{od}} x_{i,a}(t) \leq \mu_a \quad \forall a \in A, \forall t \in T, \quad (6)$$

$$x_{i,a}(t) = \sum_{r \in \mathcal{R}_{od}} f_{i,r}(t + T_{a,r}) \delta_{a,r}(o,d) \quad \forall a \in A, \forall t \in T, \forall i \in \mathcal{N}_{od} \forall od \in \mathcal{W}, \quad (7)$$

$$f_{i,r}(t), x_{i,a}(t) \in \{0, 1\} \quad \forall a \in A, \forall r \in \mathcal{R}_{od}, \forall t \in T, \forall i \in \mathcal{N}_{od}, \forall od \in \mathcal{W}, \quad (8)$$

where $f_{i,r}(t)$ denotes the allocation of a bundle of permits to user i and $x_{i,a}(t)$ denotes the allocation of a network permit to user i . Specifically, $f_{i,r}(t)$ is 1 if user i is allocated a bundle of permits for a set of links required to travel along path r and to arrive at time period t and is zero otherwise. Hence, $x_{i,a}(t)$ is 1 if user i is allocated a network permit for link a in time period t and is zero otherwise.

This is the combinatorial optimization problem of finding an efficient network permit allocation pattern $(\mathbf{f}^*, \mathbf{x}^*)$, subject to the physical constraints on flows representing the network performance. The first constraint (5) is the condition that each user makes at most one trip in the interval $[0, I]$. The second constraint (6) is the capacity constraint on each link. The third constraint (7) expresses the flow conservation between link flows and path flows for each user; that is, the link flow $x_{i,a}(t)$ entering into link a at time period t is the sum of the flows on all paths going through that link and arriving at the destination at time $t + T_{a,r}$. The travel time required for arriving at the destination from the upstream node k (of the link a) through path r (containing link a) is given by:

$$T_{a,r} = \sum_{a' \in A} t_{a'} \delta_{a',r}(k,d), \quad (9)$$

where $\delta_{a',r}(k,d)$ is a typical element of the path-link incidence matrix for node pair (k, d) .

Although the road manager seeks to solve the problem [DSO] to achieve the system optimal permit allocation pattern, solving the problem directly poses two major difficulties: (i) the objective function of the problem includes users' private valuations and (ii) the problem is NP-hard (i.e., no polynomial-time algorithm exists for it). The first difficulty comes from the obvious fact that the manager cannot accurately obtain such private information. The second difficulty comes from the fact that the problem [DSO] is an integer multicommodity flow problem.

One possible way to address these difficulties might be to apply conventional combinatorial auctions to this problem. For example, the VCG mechanism can overcome the first difficulty, at least in principle, because it gives users an incentive (Vickrey payment) to report their valuations truthfully (i.e., strategy-proofness). However, the VCG mechanism cannot overcome the second difficulty, because the problem [DSO] must be solved *exactly* to determine the optimal permit allocation and to compute the Vickrey payments (i.e., a computationally infeasibility). One natural approach to handling the problem is to seek a sub-optimal solution instead of the optimal solution. However, the VCG mechanism allowing non-optimal allocations is not strategy-proof, as each user has an incentive to bid false valuations to increase the utility (Nisan and Ronen, 2007). Therefore, it is difficult to apply the VCG mechanism directly to the trading markets.

5. Day-to-day auction mechanism: an auction mechanism with day-to-day capacity control

In this section, we propose a novel auction mechanism including a day-to-day capacity control, which is readily implementable for general networks. We call this mechanism the *day-to-day auction mechanism*. To avoid computational infeasibility such as that in the case of the VCG mechanism, the proposed mechanism employs an evolutionary approach. Although the evolutionary approach cannot be employed for the one-shot auctions that are typically treated in auction theory, it can be utilized for a tradable network permits scheme in which the auction is opened to morning commuters each day.

Before describing the proposed mechanism, we introduce some modifications of the model. In the proposed mechanism, we consider time-dependent permit allocation patterns and their day-to-day dynamics. We then denote the day by $s \in S$. Suppose that each user behaves myopically and makes one's own choice so as to maximize the following utility defined for each day s :

$$u_{i,r}^s(t) \equiv v_{i,r}(t) - P_r^s(t). \quad (10)$$

This implies that the user considers only one's own allocation of the bundles and payment on each day, so the user's true valuations are constant for all days.

5.1. Reformulation of the DSO problem with path capacities and the Benders decomposition principle

The day-to-day auction mechanism is based on the idea of reformulating the problem [DSO] as a problem with *non-individual* variables and decomposing it into two problems, a master problem and a sub-problem, based on then applying the Benders decomposition principle (e.g., Benders, 1962; Lasdon, 1970). We then solve these problems on day-to-day basis. Further, in order to achieve an efficient permit allocation with imperfect information about users, the mechanism also exploits an auction mechanism for solving the sub-problem.

We let $F_r(t), X_a(t) \in \mathbb{Z}_+$ denote a non-individual path variable and a non-individual link variable, respectively. By using these variables, the problem [DSO] with non-individual variables is formulated as

$$\max_{(\mathbf{f}, \mathbf{F}, \mathbf{X})} \text{SS}(\mathbf{f}, \mathbf{F}) \equiv \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \sum_{t \in T} \sum_{r \in R_{od}} v_{i,r}(t) f_{i,r}(t), \quad (11)$$

$$\text{s.t.} \quad \sum_{t \in T} \sum_{r \in R_{od}} f_{i,r}(t) \leq 1 \quad \forall i \in \mathcal{N}_{od}, \forall od \in W, \quad (12)$$

$$\sum_{i \in \mathcal{N}_{od}} f_{i,r}(t) \leq F_r(t) \quad \forall r \in R_{od}, \forall t \in T, \forall od \in W, \quad (13)$$

$$X_a(t) \leq \mu_a \quad \forall a \in A, \forall t \in T, \quad (14)$$

$$X_a(t) = \sum_{od \in W} \sum_{r \in R_{od}} F_r(t + T_{a,r}) \delta_{a,r}(o,d) \quad \forall a \in A, \forall t \in T, \quad (15)$$

$$f_{i,r}(t) \in \{0, 1\}, F_r(t), X_a(t) \in \mathbb{Z}_+ \quad \forall a \in A, \forall r \in R_{od}, \forall t \in T, \forall i \in \mathcal{N}_{od}, \forall od \in W. \quad (16)$$

Each non-individual path variable $F_r(t)$ in Eq. (13) is interpreted as a *path capacity* that is the number of bundles of permits sold for the path. Constraint (12) is the condition that each user makes at most one trip. Constraint (13) is the path capacity constraint on each path. Constraints (14) and (15) are the conditions that the path capacity satisfies constraints stemming from link capacities.

This problem includes two types of variables, individual variables \mathbf{f} and non-individual variables (\mathbf{F}, \mathbf{X}) , and naturally becomes a bi-level problem based on the Benders decomposition principle:

$$\max_{(\mathbf{F}, \mathbf{X})} \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \sum_{t \in T} \sum_{r \in R_{od}} v_{i,r}(t) f_{i,r}(\mathbf{F}(t)), \quad (17)$$

s.t. Eqs. (14) and (15), and $F_r(t), X_a(t) \in \mathbb{Z}_+$,

where $\mathbf{f}(\mathbf{F})$ is an optimal solution of the following problem for a parameter \mathbf{F} :

$$\max_{\mathbf{f} \geq \mathbf{0}} \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \sum_{t \in T} \sum_{r \in R_{od}} v_{i,r}(t) f_{i,r}(t), \quad (18)$$

s.t. Eqs. (12) and (13).

The upper level problem (master problem) determines the optimal path capacity that maximizes the social surplus. The lower level problem (sub-problem) determines the efficient allocation of bundles of permits under the condition that each path capacity is fixed. Note that the sub-problem reduces to independent sub-problems in terms of OD pairs because path capacities differ among OD pairs. Furthermore, the sub-problem (18) is the *Hitchcock transportation problem* and so a linear relaxation of the sub-problem satisfies *total unimodularity* (e.g., Papadimitriou and Steiglitz, 1982). Thus, we can obtain an integer solution by solving a linear relaxation of the sub-problem, because the path capacities are integer valued.

To demonstrate a clear relationship between the master problem and the sub-problem, we consider the following dual problem of the sub-problem:

$$Z(\mathbf{F}) \equiv \min_{(\pi, \mathbf{P}) \geq \mathbf{0}} \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \pi_i + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r(t) P_r(t), \quad (19)$$

$$\text{s.t.} \quad \pi_i \geq v_{i,r}(t) - P_r(t) \quad \forall r \in R_{od}, \forall t \in T, \forall i \in \mathcal{N}_{od}, \forall od \in W, \quad (20)$$

where (π, \mathbf{P}) are Lagrange multipliers for constraints (12) and (13). As shown in Section 6.1, the multipliers are equal to the user payoffs and competitive equilibrium bundle prices that are realized in an auction (we call these variables *demand information*). From the duality theorem, the optimal value of the objective function (19) coincides with the optimal value of the objective function (18); that is,

$$Z(\mathbf{F}) = \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \pi_i(\mathbf{F}) + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r(t) P_r(\mathbf{F}(t)) = \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \sum_{t \in T} \sum_{r \in R_{od}} v_{i,r}(t) f_{i,r}(\mathbf{F}(t)), \quad (21)$$

where $(\pi(\mathbf{F}), \mathbf{P}(\mathbf{F}))$ is an optimal solution of the dual problem (19) for a parameter \mathbf{F} . Hence, $(\pi(\mathbf{F}), \mathbf{P}(\mathbf{F}))$ is an extreme point of the convex feasible region Ω_{SD} that consists of the constraints (20) and nonnegative constraints. By using the function (21), we can transform the master problem into the following problem:

$$\max_{(\mathbf{F}, \mathbf{X})} Z(\mathbf{F}) = \sum_{od \in Wi \in \mathcal{N}_{od}} \pi_i(\mathbf{F}) + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r(t) P_r(\mathbf{F}(t)), \quad (22)$$

$$= \max_{(\mathbf{F}, \mathbf{X})} \left[\min_{(\pi^s, \mathbf{P}^s) \in V} \sum_{od \in Wi \in \mathcal{N}_{od}} \pi_i^s + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r(t) P_r^s(t) \right], \quad (23)$$

s.t. Eqs. (14) and (15), and $F_r(t), X_a(t) \in \mathbb{Z}_+$,

where V is the finite set of all extreme points of the convex feasible region Ω_{SD} . From this formulation, we see that path capacities are adjusted on the basis of the demand information. Moreover, this problem is equivalent to the following problem:

$$\max_{\theta \geq 0, (\mathbf{F}, \mathbf{X})} \theta, \quad (24)$$

s.t. Eqs. (14) and (15), $F_r(t), X_a(t) \in \mathbb{Z}_+$,

$$\theta \leq \sum_{od \in Wi \in \mathcal{N}_{od}} \pi_i^s + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r(t) P_r^s(t) \quad \forall (\pi^s, \mathbf{P}^s) \in V. \quad (25)$$

Problem (24) is equivalent to the problem [DSO] (with non-individual variables) if all the extreme points are known. However, it is difficult to obtain the extreme points in advance, because the number of extreme points is generally too large. Hence, we consider a relaxation problem (24) that has a subset of the extreme points in V and produces an upper bound on the optimal objective value of the problem [DSO]. This relaxed problem is called the *restricted master problem* [RMP]. We then employ an iterative approach by adding an extreme point to the problem [RMP] to improve the upper bound. Note that an extreme point is generated by solving the problem (19) for fixed path capacities \mathbf{F} .

The procedure of the proposed mechanism corresponds to solving the above two problems, iteratively. One of the greatest differences between the standard Benders decomposition and the proposed mechanism is whether or not coefficient parameters \mathbf{v}_i (i.e., truthful valuations of each user) are initially given. As mentioned in Section 4, the manager cannot observe such private information. Nevertheless, the proposed mechanism can obtain the demand information by exploiting an auction mechanism for solving the sub-problem.

5.2. Interpretation as auction mechanism with day-to-day capacity control

The day-to-day auction mechanism comprises an *auction phase* and a *path capacity adjustment phase*; the two phases are repeated on a day-to-day basis (Fig. 1). In the *auction phase* corresponding to the sub-problem, the manager sells bundles of permits to the users through an ascending auction under the condition that each capacity is fixed. In addition, the bundle prices are determined during the ascending auction so as to maximize each user’s payoff. In the *path capacity adjustment phase* corresponding to the restricted master problem, the manager adjusts each path capacity to an appropriate level by considering the demand information that was determined in the previous auction phases. Hence, the procedure of the day-to-day auction mechanism can be summarized as follows (more details of the mechanism and its properties can be found in Section 6):

1. *Initial setting.* Set $s = 1$. Determine the initial path capacities \mathbf{F}^1 . Start with a set of extreme points $V^1 = \{\emptyset\}$ and a convergence threshold $\underline{\theta}^1 = \infty$.
2. *Auction phase* (Section 6.1). For fixed path capacities \mathbf{F}^s , the manager sells bundles of permits through an ascending auction. The user payoffs and the bundle prices (π^s, \mathbf{P}^s) are also determined. If the social surplus SS^s achieved in the ascending auction is equal to or greater than the convergence threshold $\underline{\theta}^s$ that is defined in Section 6.2, then stop. Otherwise go to Step 3.
3. *Path capacity adjustment phase* (Section 6.2). Add an extreme point to the set; i.e., $V^{s+1} \equiv \{V^s \cup (\pi^s, \mathbf{P}^s)\}$. Produce the path capacities \mathbf{F}^{s+1} by solving the problem [RMP] and update the convergence threshold $\underline{\theta}^{s+1}$. Let $s = s + 1$. Go to Step 2.

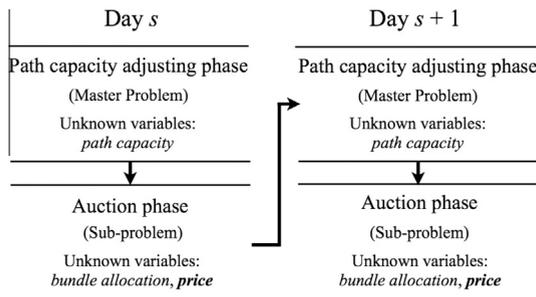


Fig. 1. Procedure of the proposed mechanism.

Note that *stop* in Step 2 means that optimal path capacities are obtained. Therefore, once the above procedure stops, the manager no longer adjusts the path capacities and sells bundles through the auction with the same optimal path capacities each day.

6. Details and properties of proposed mechanism

This section presents the details and properties of each phase of the day-to-day auction mechanism. Section 6.1 gives a detailed explanation of the auction phase. Section 6.2 gives the detailed path capacity adjustment rule. Section 6.3 analyzes the proposed overall mechanism combining two phases and proves that the day-to-day dynamics of the network allocation pattern converges to the dynamic system optimal allocation when the number of users is large.

6.1. Auction phase

In the auction phase, the manager sells bundles of permits to the users through an ascending auction for fixed path capacities. The procedure of this ascending auction corresponds to solving the sub-problem using a primal–dual algorithm. Thus, we first analyze solutions of both the sub-problem and its dual.

6.1.1. Competitive equilibrium in tradable network permit markets

Let \mathbf{F}^s be the path capacities as determined in the path capacity adjustment phase on day $s - 1$. Then, the sub-problem for each OD pair is given by the following linear program:

$$\begin{aligned} SS_{od}^s \equiv \max_{\mathbf{f} \geq \mathbf{0}} \quad & \sum_{i \in \mathcal{N}_{od}} \sum_{t \in T} \sum_{r \in R_{od}} v_{i,r}(t) f_{i,r}^s(t), \\ \text{s.t.} \quad & \text{Eqs. (12) and (13)}. \end{aligned} \quad (26)$$

The necessary and sufficient optimality conditions of the problem are given by the following Kuhn–Tucker conditions:

$$\begin{cases} \pi_i^s = v_{i,r}(t) - P_r^s(t) & \text{if } f_{i,r}^s(t) = 1 \\ \pi_i^s \geq v_{i,r}(t) - P_r^s(t) & \text{if } f_{i,r}^s(t) = 0 \end{cases} \quad \forall r \in R_{od}, \forall t \in T, \forall i \in \mathcal{N}_{od}, \quad (27)$$

$$\begin{cases} \sum_{i \in \mathcal{N}_{od}} f_{i,r}^s(t) = F_r^s(t) & \text{if } P_r^s(t) > 0 \\ \sum_{i \in \mathcal{N}_{od}} f_{i,r}^s(t) \leq F_r^s(t) & \text{if } P_r^s(t) = 0 \end{cases} \quad \forall r \in R_{od}, t \in T, \quad (28)$$

$$\pi_i^s, P_r^s(t) \geq 0 \quad \forall r \in R_{od}, \forall t \in T, \forall i \in \mathcal{N}_{od}, \forall od \in W. \quad (29)$$

Note that the allocation variables \mathbf{f} are integer because each sub-problem (26) satisfies total unimodularity. The solution $(\mathbf{f}^s, \pi^s, \mathbf{P}^s)$ consists of a competitive equilibrium allocation, payoffs and prices, respectively. In the competitive equilibrium, each user acquires the bundle of permits that maximizes one's own utility (i.e., (27)) for the given set of competitive equilibrium prices that satisfy the market clearing condition (28). Further, all users who acquire bundles have nonnegative payoffs (i.e., the user's willingness-to-pay is greater than the price requires), which is consistent with conventional traffic assignments with elastic demand.

The concept of competitive equilibrium for indivisible items is a natural extension of the classical economic concept for divisible items. Here the necessary and sufficient condition for the existence of this competitive equilibrium is that the optimal solution to the linear relaxation of the sub-problem is integer (Bikhchandani and Mamer, 1997). In addition, it has been shown that the competitive equilibrium, if it exists, is efficient (Bikhchandani and Ostroy, 2002). This can be summarized as follows:

Lemma 1. *In the tradable network permit markets on day s , there always exists a competitive equilibrium that provides an efficient network permit allocation pattern for a fixed path capacity.*

Proof. See Bikhchandani and Mamer (1997) and Bikhchandani and Ostroy (2002). \square

Note that the set of competitive equilibrium prices discussed above does not necessarily satisfy strategy-proofness. However, Leonard (1983) showed that *minimal* competitive equilibrium prices such that the payment for each user is equal to the decrease in the value of the social surplus by adding the user to the auction are equivalent to Vickrey payments that produce strategy-proofness. In addition, Leonard (1983) formulated the problem of finding the minimal competitive equilibrium prices:

$$\min_{(\pi^s, \mathbf{P}^s) \geq \mathbf{0}} \sum_{t \in T} \sum_{r \in R_{od}} F_r^s(t) P_r^s(t), \quad (30)$$

s.t. Eq. (20),

$$\sum_{i \in N_{od}} \pi_i^s + \sum_{t \in T} \sum_{r \in R_{od}} F_r^s(t) P_r^s(t) = SS_{od}^s. \quad (31)$$

This problem minimizes competitive equilibrium prices (or maximizes user payoffs) subject to the conditions that the solution of this problem also solves the dual of the sub-problem.

From the above discussions, we find that the sub-problem can be solved through the VCG mechanism in a computationally efficient manner: the allocation problem (26) is merely the transportation problem, and Vickrey payments are computed by solving only one linear program (30). However, there remains the problem of complication of the bidding rule: users have to report sealed bids giving their value of all bundles of permits. This bidding rule is also undesirable in terms of the privacy, as users are required to reveal more of their private information than is necessary.

6.1.2. Ascending proxy auction

The proposed mechanism employs an ascending auction to resolve the problems of sealed bid auctions and to produce outcomes (i.e., allocation and prices of bundles of permits) in an informationally efficient manner. More specifically, we employ the (exact) ascending auction proposed by Demange et al. (1986) (we call this the *DGS auction*). In this auction, users report only the “names” of the bundles of permits in which they are interested. The procedure of the DGS auction corresponds to solving the sub-problem using a primal–dual algorithm, which is described as follows (see also Bikhchandani et al., 2002):

1. *Initialization.* Set $\mathbf{P}^s = \mathbf{0}$ for all bundles.
2. *Bidding phase.* Each user reports the names of the bundles that maximize one’s own payoff under the current prices \mathbf{P}^s ; i.e., a demand set $D_i(\mathbf{P}^s) \equiv \arg \max_{r,t} [v_{i,r}(t) - P_r^s(t)]$. If each user can be allocated a bundle from the user’s demand set, then stop because \mathbf{P}^s are equilibrium prices. Otherwise go to Step 3.
3. *Price adjustment phase.* The manager chooses a *minimal overdemanded set* $M(\mathbf{P}^s)$ and raises the prices of the bundles in that set (i.e., $P_r^s(t) = P_r^s(t) + 1, \forall (r, t) \in M(\mathbf{P}^s)$). Go to Step 2.

Here, an overdemanded set is a set of bundles for which the number of users demanding only bundles in that set exceeds the number of bundles sold in the auction, and the minimal overdemanded set is an overdemanded set of bundles with no proper overdemanded subset.

In the DGS auction, the prices of the bundles converge to the minimal competitive equilibrium prices if each user reports the demand set truthfully (i.e., a myopic best response strategy) because the *minimal overdemanded set* is chosen in Step 3. Hence, the outcome of the DGS auction is equal to the VCG outcome. Further, the truthful reporting of the demand set constitutes a Nash equilibrium for each user in each Step 2.

In a practical implementation of the DGS algorithm, it is hard for each user to report the demand set in each bidding phase; i.e., the transaction cost is too large. We therefore introduce a proxy agent system to support the bidding of users. Proxy systems are popular and have been installed in many Internet auctions (e.g., eBay and Yahoo). Under such a system, each user reports valuations to a proxy agent for some bundles that interest the user. Then, the proxy agent bids in the auction on the basis of the information received from the user. This system not only reduces the transaction cost of the bidding phase, but also prevents strategic behaviors (e.g., a non-myopic best response strategy) in each bidding phase.

Let us now introduce the proxy agent system proposed by Parkes and Ungar (2000) into the DGS auction. Step 1 and Step 2 are then modified as follows:

- 1'. Before starting the auction, each user reports information of valuations for some bundles to one’s own proxy agent. Set $\mathbf{P}^s = \mathbf{0}$ for all bundles.
- 2'. Based on the information received and the current prices, each proxy agent submits each user demand set $D_i(\mathbf{P}^s)$. If each user can be allocated a bundle from one’s own demand set, then stop because the \mathbf{P}^s are equilibrium prices. Otherwise, go to Step 3.

In Step 2', the user needs to update information if the proxy agent does not have enough information to submit the demand set. Since the proxy DGS auction restricts user strategies (in each bidding phase) to a myopic best response strategy, a dominant strategy is truthful reporting of the valuations to the proxy agent. From what has been discussed above and Lemma 1, we obtain the following proposition:

Proposition 1. *The network permits allocation pattern achieved under the proxy DGS auction for implementing the tradable network permit markets on day s is efficient, and the prices of bundles of permits converge to the minimal competitive equilibrium prices. A dominant strategy for each user is truth reporting of the valuations of bundles to the proxy agent.*

Proof. See Demange et al. (1986) and Parkes and Ungar (2000). \square

6.2. Path capacity adjustment phase

In the path capacity adjustment phase, the road manager first generates the demand information (i.e., payoffs and prices). The prices \mathbf{P}^s can be obtained directly in the auction phase for all OD pairs. The payoffs π^s , however, are computed indirectly. In the proxy DGS auction, since each user reports the user's true valuations for interesting bundles to the proxy agent, the manager can obtain the user's winning valuation $v_{i,r}^*(t)$. Then, the manager calculates a total payoff Π^s from the duality theorem:

$$\Pi^s \equiv \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \pi_i^s = \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} v_{i,r}^*(t) - \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r^s(t) P_r^s(t). \quad (32)$$

Note that the manager needs to know only the total payoff to adjust path capacities.

After generating the demand information, the manager considers all demand information for the current day and past days, $V^{s+1} \equiv \{V^s \cup (\Pi^s, \mathbf{P}^s)\}$, and adjusts each path capacity by solving the restricted master problem [RMP]. However, this is computationally intensive because the problem [RMP] (i.e., the problem (24)) is a large integer programming (IP) problem with one continuous variable. To avoid this, we solve the linear relaxation of the problem [RMP] and obtain an integer solution by rounding off the fractional solution. Such a strategy was suggested by McDaniel and Devine (1977) and has successfully used in various problems (e.g., Cordeau et al., 2000). This strategy is suitable for our situation because non-individual variables (path capacities) in the problem [RMP] are control variables of the road manager and can be treated as continuous variables, although the individual variables (allocation of network permits) cannot be treated as continuous. In addition, note that the relaxation of integrality constraints does not affect the convex feasible region Ω_{SD} of the dual sub-problem and that an extreme point can be generated from any integer solution. Thus, the problem with continuous variables $(\tilde{\mathbf{F}}, \tilde{\mathbf{X}})$ that the road manager needs to solve is given as

$$\max_{\tilde{\theta} \geq 0, (\tilde{\mathbf{F}}^{s+1}, \tilde{\mathbf{X}}^{s+1}) \geq \mathbf{0}} \tilde{\theta}, \quad (33)$$

$$\text{s.t. } \tilde{\theta} \leq \Pi^s + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} \tilde{F}_r^{s+1}(t) P_r^s(t) \quad (\Pi^s, \mathbf{P}^s) \in V^{s+1}, \quad (34)$$

$$\tilde{X}_a^{s+1}(t) \leq \mu_a \quad \forall a \in A, \quad \forall t \in T, \quad (35)$$

$$\tilde{X}_a^{s+1}(t) = \sum_{od \in W} \sum_{r \in R_{od}} \tilde{F}_r^{s+1}(t + T_{a,r}) \delta_{a,r(o,d)} \quad \forall a \in A, \quad \forall t \in T. \quad (36)$$

The optimal objective value is an upper bound on the maximum social surplus SS^* of the problem [DSO], which is weaker than the upper bound that is produced with the integer programming problem [RMP].

From the optimality conditions of the problem, a path capacity adjustment rule can be derived as

$$\begin{cases} \sum_{a \in A} p_a(t - T_{a,r}) \delta_{a,r(o,d)} = \bar{P}_r(t) & \text{if } \tilde{F}_r^{s+1}(t) > 0 \\ \sum_{a \in A} p_a(t - T_{a,r}) \delta_{a,r(o,d)} \geq \bar{P}_r(t) & \text{if } \tilde{F}_r^{s+1}(t) = 0 \end{cases} \quad \forall r \in R_{od}, \quad \forall t \in T, \quad \forall od \in W, \quad (37)$$

where the $\bar{\mathbf{P}}$ are the (convex combinations of) bundle prices that produce the weak upper bound (i.e., the constraint (34) is bounded), and \mathbf{p} is the Lagrange multiplier for the constraint (35). This Lagrange multiplier is interpreted as a permit price for each link that satisfies the following (market clearing) condition:

$$\begin{cases} \tilde{X}_a^{s+1}(t) = \mu_a & \text{if } p_a(t) > 0 \\ \tilde{X}_a^{s+1}(t) \leq \mu_a & \text{if } p_a(t) = 0 \end{cases} \quad \forall a \in A, \quad \forall t \in T. \quad (38)$$

If the path capacity is positive in the path capacity adjustment rule (37), the bundle price estimated for the path by means of link permit prices and is equal to the bundle price determined in the auction phase. For a path whose the estimated price exceeds the realized price, the path capacity is zero. This means that no path capacities are allocated to the worthless paths. The integer path capacities \mathbf{F}^{s+1} on day $s+1$ can be obtained by rounding-off all continuous path capacities; i.e., $F_r^{s+1}(t) = \lfloor \tilde{F}_r^{s+1}(t) \rfloor$.

6.2.1. Stabilizing strategy for the Benders decomposition

Although the problem (33) is easy to solve, there remains one issue relevant to the convergence rate of the Benders decomposition; i.e., path capacities usually oscillate, which results in slow convergence (Magnanti and Wong, 1981). To accelerate and stabilize the Benders decomposition, we add *boxstep constraints* (Marsten et al., 1975) to the above problem (33):

$$\tilde{F}_r^s(t) - \epsilon \leq \tilde{F}_r^{s+1}(t) \leq \tilde{F}_r^s(t) + \epsilon \quad \forall r \in R_{od}, \quad \forall t \in T, \quad \forall od \in W, \quad (39)$$

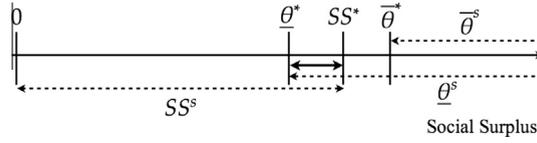


Fig. 2. Relationship between the convergence threshold, the weak upper bound, the achieved social surplus, and the maximum social surplus.

where ϵ is a boxstep parameter. At each step, the solution $\tilde{\mathbf{F}}^{s+1}$ to the master problem is constrained to lie within a box centered on the previous solution $\tilde{\mathbf{F}}^s$ and so the oscillation is dramatically reduced. Note that the problem including the boxstep constraints does not necessarily produce an upper bound on the maximum social surplus SS^* . Thus, we solve the problem (33) to obtain the upper bound $\bar{\theta}$.

6.3. Convergence of the day-to-day auction mechanism

We now establish the convergence result of the day-to-day auction mechanism on the basis of the Benders decomposition technique. The standard Benders decomposition algorithm converges to an optimal solution when the strong upper bound obtained by the problem [RMP] is equal to the optimal objective value of the sub-problem (i.e., the social surplus achieved in the auction phase). However, the weak upper bound $\bar{\theta}$ obtained with the proposed mechanism will exceed the maximum value of the social surplus SS^* even if all the extreme points are generated, and thus we cannot use $\bar{\theta}$ as the convergence threshold.

To resolve this problem, we introduce a new convergence threshold $\underline{\theta}$:

$$\underline{\theta} \equiv \min_{(II^s, \mathbf{P}^s) \in V^{s+1}} \cdot II^s + \sum_{t \in I} \sum_{r \in R} F_r^{s+1}(t) P_r^s(t), \tag{40}$$

and an update rule of the threshold is

$$\underline{\theta}^{s+1} = \min \{ \underline{\theta}^s, \underline{\theta} \}. \tag{41}$$

The criterion based on $\underline{\theta}$ optimizes (i.e., minimizes) the objective function of the problem [RMP] only with respect to extreme points (II^s, \mathbf{P}^s) given at the integer path capacities \mathbf{F}^{s+1} , which results in good convergence properties as shown in the proof of the proposition below. The threshold $\underline{\theta}$ is equal to or less than the strong upper bound, since it does not maximize the objective function of the problem [RMP] with respect to the path capacities. Therefore, we conclude that the permit allocation under the proposed mechanism converges to an approximate dynamic system optimal state when the achieved social surplus $SS^{s+1} (= \sum_{od} SS_{od}^{s+1})$ in the auction phase is equal to or greater than the convergence threshold $\underline{\theta}^{s+1}$.

Fig. 2 shows the relationship between the convergence threshold $\underline{\theta}^s$, the weak upper bound $\bar{\theta}^s$, the achieved social surplus SS^s , and the maximum social surplus SS^* . The horizontal axis represents the social surplus (or its upper bound) and dotted lines represent the ranges in which the variables can exist. The achieved social surplus SS^s can exist in the range $[0, SS^*]$. The convergence threshold and the weak upper bound take minimum values $\underline{\theta}^*$ and $\bar{\theta}^*$ when we have all the extreme points.

By using the convergence threshold $\underline{\theta}^s$, we obtain the social surplus in the range that is represented by the solid arrow in Fig. 2. The ratio SS^s/SS^* between the achieved social surplus and the maximum value of the social surplus is confined within in the range

$$\frac{\underline{\theta}^s}{SS^*} \leq \frac{SS^s}{SS^*} \leq 1. \tag{42}$$

Assuming that the ratio between the total number of users $Q (= \sum_{od} |\mathcal{N}_{od}|)$ and the total link capacity $\sum_t \sum_a \mu_a$ is held constant, the range Eq. (42) converges to zero (i.e., the left-hand side of (42) converges to 1) when the number of users is sufficiently large. This is because the effect of rounding off the continuous path capacities is negligible in that case. In addition, a new extreme point is generated in each auction phase when the achieved social surplus does not satisfy the convergence criterion, so the proposed mechanism can converge in a finite number of steps. Therefore, the following proposition holds.

Proposition 2. Assume that the ratio between the number of users and total link capacity is constant. Then, the day-to-day auction mechanism converges in a finite number of steps, and the value of the social surplus achieved by the mechanism reaches its maximum value when the number of users is large.

Proof. See Appendix A for the proof. \square

7. An extended mechanism which obviates path enumeration

The day-to-day auction mechanism presented in the previous sections assumes that the road manager can enumerate all the paths that users may choose. However, it is not necessarily evident how the manager should do so for large-scale net-

works. To obviate path enumeration, we construct an extended mechanism by introducing a path generation phase into the day-to-day auction mechanism. This consists of applying a column generation procedure to the system optimal allocation problem [DSO]. In the extended mechanism, users generate paths successively, and hence path enumeration is obviated for the manager.

A column generation for a network flow problem considers a problem that has only a subset of the paths of the original problem (i.e., a *restricted master problem*) and paths are generated as needed (Ahuja et al., 1993). Hence, by considering only a subset of the (dynamic) paths of the problem [DSO], a restricted master problem [C-RMP] is formulated as

$$\max_{(\mathbf{f}, \mathbf{x})} \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \sum_{t \in T} \sum_{r \in R_{od}(t)} v_{i,r}(t) f_{i,r}(t), \quad (43)$$

$$\text{s.t.} \quad \sum_{t \in T} \sum_{r \in R_{od}(t)} f_{i,r}(t) \leq 1 \quad \forall i \in \mathcal{N}_{od}, \quad \forall od \in W, \quad (44)$$

$$\sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} x_{i,a}(t) \leq \mu_a \quad \forall a \in A, \quad \forall t \in T, \quad (45)$$

$$x_{i,a}(t) = \sum_{t \in T} \sum_{r \in R_{od}(t)} f_{i,r}(t + T_{a,r}) \delta_{a,r(o,d)} \quad \forall a \in A, \quad \forall t \in T, \quad \forall i \in \mathcal{N}_{od} \quad \forall od \in W, \quad (46)$$

$$f_{i,r}(t), x_{i,a}(t) \in \{0, 1\} \quad \forall a \in A, \quad \forall r \in R_{od}(t), \quad \forall t \in T \quad \forall i \in \mathcal{N}_{od}, \quad \forall od \in W, \quad (47)$$

where $R_{od}(t)$ is a subset of paths in destination arrival time period t . Since the problem [C-RMP] and the problem [DSO] have the same optimization problem except for the number of paths, we can solve the problem [C-RMP] through the day-to-day auction mechanism presented in the previous sections.

A new path is generated by solving a *column generation sub-problem* corresponding to the *pricing step* of the simplex algorithm (for the linear relaxation of the problem [C-RMP]). In the standard column generation for a multicommodity flow problem, the sub-problem is given as a shortest path problem for each commodity (Ahuja et al., 1993). Thus, by following the standard theory, our sub-problem is formulated as the following all-or-nothing problem for each user:

$$\pi_i^* \equiv \max_{\mathbf{f}_i \geq \mathbf{0}} \sum_{t \in T} \sum_{r \in R_{od}(t)} \left[v_{i,r}(t) - \sum_{a \in A} \hat{p}_a(t - T_{a,r}) \delta_{a,r(o,d)} \right] f_{i,r}(t), \quad (48)$$

$$\text{s.t.} \quad \sum_{t \in T} \sum_{r \in R_{od}(t)} f_{i,r}(t) \leq 1, \quad (49)$$

where $\hat{p}_a(t)$ is an optimal Lagrange multiplier for the link capacity constraint (45) of the linear relaxation of the restricted master problem [C-RMP], which is interpreted as an optimal link permit price. These link permit prices are obtained at the final path capacity adjustment phase of the day-to-day auction mechanism (see Section 6.2).

The column generation sub-problem yields a path that maximizes each user payoff for given constant link permit prices $\hat{\mathbf{p}}$. The path is generated if a maximum payoff exceeds the current payoff achieved in the final auction phase of the day-to-day auction mechanism. Specifically, the path is generated if the optimal value of the objective function π_i^* exceeds an optimal Lagrange multiplier $\hat{\pi}_i$ for the constraint (44); i.e., $\lambda_i \equiv \pi_i^* - \hat{\pi}_i > 0$. To improve his or her payoff, the user requests that the manager sells the bundle for the path in the auction phase. The road manager receives the requests of all users and adds the paths to the set $R_{od}(t)$ (if the path do not exists in the set). Then, the restricted master problem [C-RMP] is again solved through the day-to-day auction mechanism.

The steps in the extended mechanism mentioned above can be summarized as follows:

1. *Initial setting.* Set $n = 1$. Determine the initial path set $R_{od}^1(t)$ for each OD pair at each destination arrival time period.
2. *Day-to-day auction phase.* For a fixed path set $R_{od}^n(t)$, the restricted master problem [C-RMP] is solved through the day-to-day auction mechanism (see Sections 5 and 6). The optimal link permit prices $\hat{\mathbf{p}}^n$ are determined in the final path capacity adjustment phase and are announced by the road manager.
3. *Path generation phase.* Each user finds a path by solving the column generation sub-problem and requests that the manager adds the path if the maximum payoff π_i^* exceeds the current payoff $\hat{\pi}_i^n$. If all requested paths exist in the path set $R_{od}^n(t)$, then stop. Otherwise, the road manager creates a new path set $R_{od}^{n+1}(t)$ by adding requested paths to the set $R_{od}^n(t)$. Let $n = n + 1$. Go to Step 2.

The paths are efficiently generated in Step 3 because the numerous number of users generate paths simultaneously. The road manager, on the other hand, employs a path-adding rule that allows each user to purchase not only paths generated by himself but also those generated by other users of the same OD pair,⁸ which promotes path generation. The extended mechanism is guaranteed to converge because the number of paths is finite. Furthermore, when the number of users is large, the allocation of network permits achieved under the extended mechanism converges to the optimal one (i.e., the optimal solution of the problem [DSO]) since the gap between the problem [DSO] and the linear relaxation converges to zero (Proposition 6.2).

⁸ If we employ the standard column generation procedure, subsets of the paths differ among users because the column generation sub-problem (48) is formulated for each user. However, in the auction phase, it will be more natural that the same set of paths are sold for all users of the same OD pair.

8. Numerical example

We finally show a numerical example to demonstrate the convergence properties of the proposed mechanism in a realistic network. The network that we employ is the Sioux Falls network (LeBlanc et al., 1975) which has 24 nodes and 76 links (Fig. 4). The physical conditions of each link (i.e., free-flow travel time, capacity), which is based on Han (2003), are summarized in Table B.1 in Appendix B. The network has 528 OD pairs, which was used by LeBlanc et al. (1975), and the number of users for each OD pair is a quarter of the number provided in Dr. Hillel Bar-Gera's website (<http://www.bgu.ac.il/~bargera/tntp/>); i.e., the total number of users is 90,150. We set time interval for each time period to $\Delta t = 3$ (minute) and the number of time periods to $|T| = 40$. The desired arrival time period for each user is set randomly and the distribution of the desired arrival time periods is shown in Fig. 3. Under this distribution, the network is congested (i.e., almost links have positive permit prices) during peak periods. As the initial path set for each OD pair, we simply choose some shortest paths. A box step parameter $\epsilon = 5$ is chosen. An optimal social surplus is calculated by 10,000 iterations of the proposed mechanism for a sufficiently accurate determination of the maximum one.

Let us now show the numerical results of the proposed mechanism. Fig. 5 illustrates the convergence process of the proposed mechanism until the relative error between the achieved social surplus SS^s and the optimal social surplus is reduced below 0.05%. The horizontal axis represents the number of days, s , and the vertical axis represents the ratio between the achieved social surplus SS^s on each day and the optimal social surplus. The vertical lines (at day 59, 110, 164, 232, ...) show

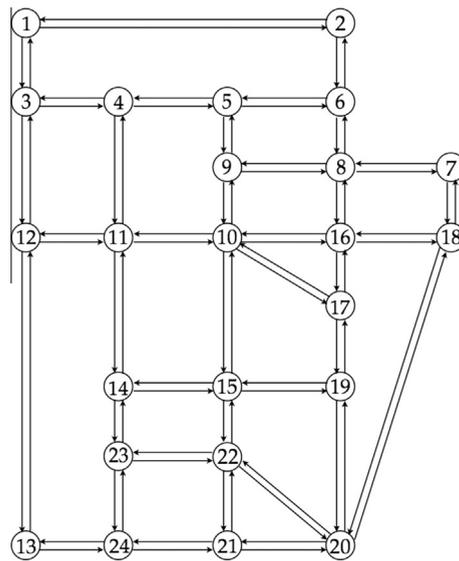


Fig. 3. Sioux Falls network.

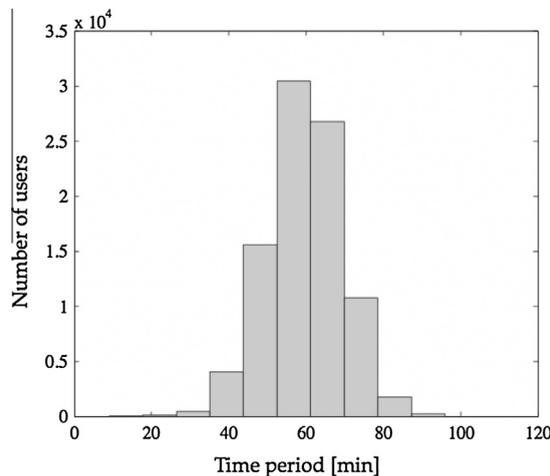


Fig. 4. Distribution of the desired arrival times.

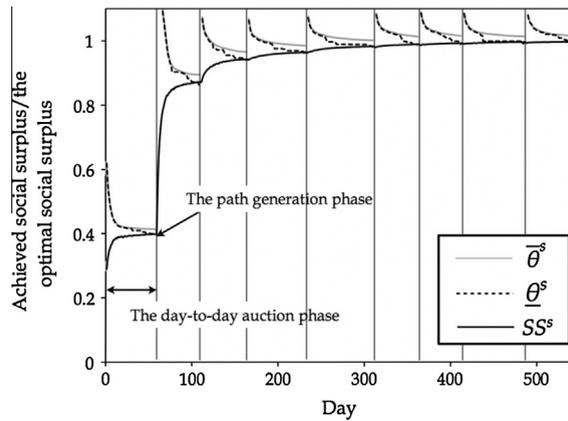


Fig. 5. Convergence process of the proposed mechanism.

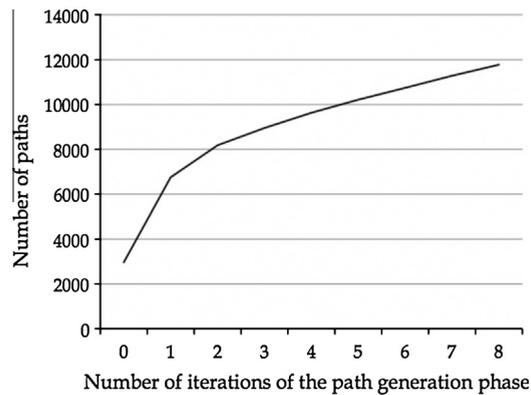


Fig. 6. Number of paths in each day-to-day auction phase.

days at which a day-to-day auction phase (or mechanism) terminated. On such a day, the path generation phase starts. Note that the path set is fixed in each day-to-day auction phase.

By using Fig. 5, we explain the convergence properties of the first day-to-day auction phase from day 1 to day 59. In this phase, the achieved social surplus SS^s (the solid black curve) increases as path capacities are adjusted on a day-to-day basis. Conversely, the upper bound of the maximum social surplus $\bar{\theta}^s$ (the gray curve) for a fixed path set and the convergence threshold $\underline{\theta}^s$ (the black dotted curve) that are obtained in the path capacity adjustment phase decrease monotonically. Eventually, these three values converge to the almost the same value. This means that the allocation of network permits achieved under the day-to-day auction phase converges to the approximate dynamic system optimal allocation for a fixed path set.

After the first day-to-day auction phase terminates (at day 59), the first path generation phase starts. In the path generation phase, each user requests a path to improve his or payoff based on the current permit prices and payoff realized in the previous day-to-day auction phase. As a result of increasing the number of paths, the achieved social surplus increases drastically in the second day-to-day auction phase. This is because a large number of paths is generated in the first path generation phase (see Fig. 6). We also see from Fig. 6 that the number of paths generated in each subsequent phase decreases, and then the achieved social surplus reaches close to the optimal value after a small number of iterations of the path generation phase.

9. Conclusion

Akamatsu et al. (2006) and Akamatsu (2007b) proposed a dynamic traffic congestion control scheme, called tradable network permits, and proved its efficiency properties for general networks. To implement trading markets for the network permits, we proposed an auction mechanism for general networks. We first discussed the impossibility of applying the VCG mechanism to the trading markets due to NP-hardness. To avoid such computational infeasibility, we constructed a day-to-day auction mechanism that is readily implementable. We then proved that the proposed mechanism is strategy-proof and that the network permit allocation pattern under this mechanism converges to an approximation of the socially optimal state in the sense that the achieved social surplus reaches its maximum value when the number of users is large. Furthermore, we showed that the proposed mechanism can be extended to obviate path enumeration by introducing a column generation procedure, and we demonstrated its convergence properties for a realistic network.

Throughout this paper, we constructed the implementation mechanism for tradable network permits, considering the first-best situation in which the road manager can issue network permits for all links. This does not necessarily implies that the proposed mechanism works effectively in the second-best situation in which the manager can issue permits for partial links. In that case, queuing congestion occurs at a link that is not controlled by the scheme. To address the case, we need to connect the tradable network permits scheme to a DTA problem; this is not a trivial problem because we would face complex interactions among queuing congestion. Nevertheless, since this direction of research increases the applicability of the scheme and its implementation mechanism, further exploration on this issue is one of the challenging but important issues that should be addressed in future work.

While this paper has focused on managing road transportation networks, the mechanism proposed seems applicable in principle to the management of other transportation networks (e.g., railway and freight networks). For example, freight networks have many users who choose routes and departure times so as to maximize their utility as is the case for road transportation networks. In contrast, the behaviors of network managers are totally different; i.e., while a road manager aims to maximize the social surplus, a freight network manager (i.e., a freight company) aims to maximize its profit. Nevertheless, managing other transportation networks using the proposed mechanism seems a fruitful topic for future work.

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Appendix A. Proof of Proposition 6.2

We first show that a new extreme point is generated in every *auction phase* until the convergence criterion is satisfied. We denote the path capacities at day s by \mathbf{F}^s and the convergence threshold by $\underline{\theta}^s$. From Eq. (40), the following holds:

$$\underline{\theta}^s \leq \Pi^s + \sum_{od \in W} \sum_{t \in I} \sum_{r \in R} F_r^s(t) P_r^s(t) \quad \forall (\Pi^s, \mathbf{P}^s) \in V^s. \quad (\text{A.1})$$

From the duality theorem, the optimal value of the objective function of the sub-problem at day s (i.e., the value of the social surplus achieved by the ascending proxy auction), SS_{od}^s , coincides with the optimal value of the objective function of its dual problem; that is

$$SS^s = \sum_{od \in W} SS_{od}^s = \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \sum_{t \in T} \sum_{r \in R_{od}} v_{i,r}(t) f_{i,r}^{s*}(t) = \Pi^{s*} + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r^s(t) P_r^{s*}(t), \quad (\text{A.2})$$

where $(\mathbf{f}^*, \Pi^{s*}, \mathbf{P}^{s*})$ is the optimal solution of the sub-problem and its dual problem. We here consider the case that the convergence criterion is not satisfied (i.e., $SS^s < \underline{\theta}^s$). Then, the following relationships are hold:

$$\sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \pi_i^{s*} + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r^s(t) P_r^{s*}(t) = SS^s < \underline{\theta}^s \leq \Pi^s + \sum_{od \in W} \sum_{t \in I} \sum_{r \in R} F_r^s(t) P_r^s(t) \quad (\forall (\Pi^s, \mathbf{P}^s) \in V^s). \quad (\text{A.3})$$

Hence, $(\Pi^{s*}, \mathbf{P}^{s*}) \neq (\Pi^s, \mathbf{P}^s) \in V^s$ is obtained; i.e., a new extreme point is generated. Since the number of extreme points is finite, we can conclude that the proposed mechanism converges in a finite number of steps.

Next, we show that the ratio $\underline{\theta}^s/SS^s$ in the left-hand side of Eq. (42) converges to 1 when the number of users is large (assuming that the ratio between the number of uses and the total link capacity is held constant). In order to show this, we prove that a ratio $\bar{\theta}^s/\bar{\theta}^*$ that is less than $\underline{\theta}^s/SS^s$ converges to 1. We denote the extreme point that minimizes the problem Eq. (40) by $(\underline{\Pi}, \underline{\mathbf{P}}) \in V$, and we denote the extreme point that produces the weak upper bound $\bar{\theta}^*$ by $(\bar{\Pi}, \bar{\mathbf{P}}) \in V$. Then the gap between $\bar{\theta}^*$ and $\underline{\theta}^*$ is investigated with the following equations:

$$\bar{\theta}^* - \underline{\theta}^* = \left(\bar{\Pi} + \sum_{od \in W} \sum_{t \in I} \sum_{r \in R} \tilde{F}_r^{s+1}(t) \bar{P}_r(t) \right) - \left(\underline{\Pi} + \sum_{od \in W} \sum_{t \in I} \sum_{r \in R} F_r(t) P_r(t) \right), \quad (\text{A.4})$$

$$\leq \left(\underline{\Pi} + \sum_{od \in W} \sum_{t \in I} \sum_{r \in R} \tilde{F}_r^{s+1}(t) P_r(t) \right) - \left(\underline{\Pi} + \sum_{od \in W} \sum_{t \in I} \sum_{r \in R} F_r(t) P_r(t) \right), \quad (\text{A.5})$$

$$< \left(\sum_{od \in W} \sum_{t \in I} \sum_{r \in R} P_r(t) \right) = (\text{the number of paths}) \times (\text{average price}). \quad (\text{A.6})$$

The second line represents the fact that the extreme points of minimizing (40) and (33) are different. The third line follows because the maximum rounded value of each path capacity is 1.

Alternatively, $\bar{\theta}^*$ can be estimated as follows:

$$\bar{\theta}^* \geq SS^* = \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} v_{i,r}^*(t) = (\text{the number of users}) \times (\text{average winning valuation}), \quad (\text{A.7})$$

Table B.1

Physical conditions of links in Sioux Falls network.

Link (upstream, downstream)	Free-flow travel time (min)	Capacity (vehicles/min)	Link (upstream, downstream)	Free-flow travel time (min)	Capacity (vehicles/min)
(1, 2) and (2, 1)	9	65	(11, 12) and (12, 11)	3	60
(1, 3) and (3, 1)	3	55	(11, 14) and (14, 11)	6	50
(2, 6) and (6, 2)	3	60	(12, 13) and (13, 12)	9	65
(3, 4) and (4, 3)	3	60	(13, 24) and (24, 13)	3	60
(3, 12) and (12, 3)	6	60	(14, 15) and (15, 14)	3	50
(4, 5) and (5, 4)	3	50	(14, 23) and (23, 14)	3	40
(4, 11) and (11, 4)	6	55	(15, 19) and (19, 15)	3	40
(5, 6) and (6, 5)	3	50	(15, 22) and (22, 15)	3	45
(5, 9) and (9, 5)	3	50	(16, 17) and (17, 16)	3	45
(6, 8) and (8, 6)	3	45	(16, 18) and (18, 16)	3	55
(7, 8) and (8, 7)	3	40	(17, 19) and (19, 17)	3	45
(7, 18) and (18, 7)	3	50	(18, 20) and (20, 18)	12	55
(8, 9) and (9, 8)	3	45	(19, 20) and (20, 19)	6	50
(8, 16) and (16, 8)	3	45	(20, 21) and (21, 20)	3	40
(9, 10) and (10, 9)	3	45	(20, 22) and (22, 20)	6	45
(10, 11) and (11, 10)	3	50	(21, 22) and (22, 21)	3	50
(10, 15) and (15, 10)	6	45	(21, 24) and (24, 21)	3	50
(10, 16) and (16, 10)	3	40	(22, 23) and (23, 22)	3	40
(10, 17) and (17, 10)	3	45	(23, 24) and (24, 23)	3	40

where $v_{i,r}^*(t)$ is the winning valuation when the social surplus is maximized. By using the above equations, the relative error between $\bar{\theta}^*$ and $\underline{\theta}^*$ is obtained as follows:

$$\frac{\bar{\theta}^* - \underline{\theta}^*}{\bar{\theta}^*} < \frac{(\text{the number of paths}) \times (\text{average price})}{(\text{the number of users}) \times (\text{average winning valuation})} < \frac{(\text{the number of paths})}{(\text{the number of users})}. \quad (\text{A.8})$$

Since the bundle prices obtained by the ascending proxy auction never exceed the truthful valuation of each user, the final inequality holds. When the number of users is large (i.e., $Q \rightarrow \infty$) with the ratio between the number of users and the total link capacity held constant, the relative error converges to zero because the number of paths is constant. Thus, the following equations hold:

$$\lim_{Q \rightarrow \infty} \frac{\bar{\theta}^*}{\underline{\theta}^*} = 1 \quad \Rightarrow \quad \lim_{Q \rightarrow \infty} \frac{\underline{\theta}^*}{SS^*} = 1. \quad (\text{A.9})$$

Hence, we can conclude that the range (42) converges to zero when the number of users is large.

Appendix B. Network data

See Table B.1.

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